Central Bank Digital Currency (CBDC) and Monetary Policy with Heterogeneous Agents*

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Abstract

We investigate the effects of central bank digital currency (CBDC) issuance in an economy where individuals can evade taxes by using cash. Our study presents a tractable model where all trades are voluntary and factoring in agent heterogeneity with unobservable idiosyncratic shocks. In the model, CBDC and cash compete as means of payment. The government has the ability to utilize CBDC to collect a labor tax, which, in our model, appears to be non-distortionary. While agents with a lower marginal utility might challenge the government's ability to finance its CBDC expenditure, we conjecture that a class of feasible policies can be identified for the optimal design of CBDC. Such policies could potentially involve higher nominal interest rates and lower inflation compared to the inflation rate associated with cash. In summary, the introduction of CBDC could enhance output and aggregate welfare by disincentivizing tax evasion.

JEL Classification: E42; E58; H21; H26
Keywords: cash; CBDC; labor tax; tax evasion; monetary policy

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1 Introduction

Recent advances in payment technologies have sparked a wave of interest in the introduction of central bank digital currency (CBDC) among central banks worldwide. At present, over 80% of central banks are engaging in CBDC-related research, with 10% having developed pilot projects (Boar et al. (2020)). One potential driving factor for central banks investigating CBDCs is the reduction of illicit activities associated with paper currency. Tax evasion, a prevalent illicit activity in many countries, is often associated with cash usage. According to estimates from the Internal Revenue Service (2019), tax evasion accounted for a tax revenue loss of around $441 billion between 2011 and 2013, representing approximately 1% of the US GDP during that period. Our paper focuses on how CBDC can address the issue of tax evasion, specifically, we study how CBDC can compete with cash as a medium of exchange and improve welfare by minimizing tax evasion.

We develop a general equilibrium model by extending the framework of Xiang (2013) and Andolfatto (2011). Our model is a simplified version of the celebrated Lagos and Wright (2005) (LW) framework without search frictions, where trade among individuals is restricted to occur in competitive markets, similar to the competitive equilibrium version in Rocheteau and Wright (2005). We include private information by incorporating heterogeneity among agents in their need for liquidity. Specifically, one type of agents experiences a higher marginal utility than the other after receiving an idiosyncratic liquidity shock. We study optimal monetary policy when only cash or CBDC or both cash and CBDC are available to agents as payment instruments. Both cash and CBDC have some distinguishing features that make them appealing as means of payments. Cash is anonymous and provides agents with the opportunity to evade taxes by hiding their cash balances.

In the paper, we define CBDC as government-issued money in digital format that can be used for retail transactions. An individual can use CBDC to purchase goods and services by opening an online account with the central bank, thereby ensuring widespread accessibility. We presume that tax evasion is unlikely with CBDC as the government can partially track the CBDC balances of agents through centralized blockchain technology. Agents can benefit from using CBDC by earning interest, akin to interest-bearing treasury securities, but must bear a fixed fee to open their CBDC accounts with the central bank. The fixed CBDC fee can be thought of as a cost that summarizes in a reduced form the cost of losing anonymity for the agents, adopting an electronic device, or working with the CBDC application (Davoodalhosseini (2021)).

In the literature, it is common practice to assume that the government possesses a lump-sum tax instrument. Optimal monetary policy is usually conducted with deflation (Friedman rule) and zero nominal interest rate; where the requisite deflation is financed by lump-sum taxes. Since agents can evade taxes with cash, this means that lump-sum taxation is not feasible. We restrict our model to voluntary trades. Optimal policy in our model is inflationary, which is intended to insure individuals against idiosyncratic liquidity shocks. The government does not have the ability to levy lump-sum taxes, but can levy a voluntary labor tax if agents use CBDC. The government can influence the relative rate of return on the two currencies by adjusting the supplies of cash and CBDC.

Our findings suggest that in a quasi-linear environment, the labor tax that can only be extracted when individuals use CBDC is non-distortionary. In fact, a positive tax on labor can be interpreted as a positive income tax, with no impact on efficiency. The Friedman rule can be optimal regardless of whether labor income is positively taxed. However, in the absence of lump-sum taxation, the Friedman rule does not implement a first-best solution. Our model also examines how the potential for tax evasion with cash imposes restrictions in the absence of a lump-sum tax instrument. A key component in our model is how tax evasion with cash imposes a restriction in the absence of a lump-sum tax instrument. To dissuade individuals from using cash, the only option for the government is positive inflation. If individuals use CBDC, then there will still be inflation, however they must be compensated with positive nominal interest rates. If CBDC has to have a higher relative rate of return than cash, then inflation in a CBDC regime must be lower than in a cash regime. This is our conjecture, as we do not have a proof. But the ingredients of our argument are fairly standard.

Our paper also provides insights into the optimal design of CBDC when tax evasion is a major concern in an economy. We argue that agents with a higher marginal utility will have the "right" incentives to use CBDC when their debt-constraint is binding. Conversely, agents with a lower marginal utility may have an incentive to misreport their types to acquire a higher money balance, and thus are more likely to prefer cash. We further explore how the implementation of CBDC can redistribute the purchasing power of agents to improve welfare. The specifics of this redistribution depend on policy parameters; individuals with lower marginal utility may be incentivized to use CBDC if they pay the fixed fee, assuming that inflation from CBDC is lower than that of cash. A lower inflation from CBDC will allow CBDC to offer a higher rate of return. Additionally, we derive the equilibrium fixed cost of CBDC that the government can collect.

1.1 Related literature

Our paper contributes to the burgeoning literature on CBDC in the context of tax evasion. Specifically, our work closely aligns with Wang (2020), Kwon et al. (2020), and Bajaj and Damodaran (2022). Wang (2020) explores CBDC design while considering tax evasion, portraying agents and the government in a dynamic game where the former is audited by the latter, with inflation dissuading agents from using cash to evade taxes. The author finds that the introduction of an interest-bearing CBDC decreases tax evasion, thereby increasing output and welfare. Kwon et al. (2020) study tax evasion and CBDC in relation to central bank independence. They introduce a proportional sales tax as a cost associated with CBDC that can potentially lead to distortion.

In contrast, the labor tax in our model is non-distortionary and is also tied to the fixed cost of CBDC. Bajaj and Damodaran (2022) examine tax evasion and the informal economy within a Lagos and Wright (2005) framework, where the government expends effort in collecting taxes. They include preference heterogeneity to characterize equilibrium conditions that may result from agents choosing multiple currencies. Unlike our paper, they assume that the government can observe all cash transactions. Other papers that exclusively study tax evasion and the shadow economy within an LW framework include Gomis-Porqueras et al. (2014), Aruoba (2021), and Lahcen (2020).

There are also several papers that examine the welfare implications of CBDC issuance. Williamson (2019) finds that CBDC can reduce crime associated with cash in an environment where banks have limited commitment. The paper posits that CBDC can also economize on the scarcity of safe collateral by paying interest, a point that aligns with our research. Davoodalhosseini (2021) illustrates that CBDC can enhance welfare when the central bank can cross-subsidize between different types of agents, a feature not possible with cash. While the author includes the concept of nonlinear interest-bearing CBDC, our study demonstrates the possible linkage between the interest rate on CBDC and its associated cost.

Several papers have examined the impact of CBDC on the banking sector and monetary policy. Assuming a perfectly competitive banking sector, Keister and Sanches (2021) find that while CBDC can promote exchange efficiency, it may also increase the funding costs of financial intermediaries and crowd out financial intermediation, thereby preventing an efficient level of investment. Andolfatto (2021) and Chiu et al. (2019) explore the impact of CBDC issuance on banking in imperfectly competitive markets. On the other hand, Whited et al. (2022) use U.S. bank data to quantify the impact of CBDC on bank lending. They find that if a CBDC pays interest, this may amplify the effect of monetary policy shocks on bank lending.

Issues around financial stability with CBDC issuance have also been studied by several authors, including Brunnermeier and Niepelt (2019), who derive equilibrium conditions in which CBDC can lead to financial stability. Similarly, Kim and Kwon (2019) use a general equilibrium model of bank liquidity provision, resembling Diamond and Dybvig (1983), to find that CBDC does not result in a credit crunch and hinder financial stability. More works that tackle this topic include Williamson (2021), Keister and Monnet (2020), and Fernández-Villaverde et al. (2021). In addition to these, a number of papers have explored the use of multiple means of payments in an LW model, such as Dong and Jiang (2010), Zhu and Hendry (2019), and Chiu and Wong (2015). Lastly, other papers that examine CBDC and monetary policy in a DSGE framework include Barrdear and Kumhof (2021), Ferrari et al. (2022), and Niepelt (2022).

The rest of the paper proceeds as follows. Section 2 describes the environment. Section 3 describes the decision-making problems of the agents. Section 4 characterizes the competitive monetary equilibrium in which cash and CBDC can either coexist or exist independently. The redistributive policy with CBDC is examined in details. Section 5 concludes.

2 Environment

The model is similar to that of Xiang (2013) and Andolfatto (2011). There is a continuum of infinitely-lived households consisting of consumer-producer pairs, distributed uniformly on the unit interval. Time is discrete and goes forever, indexed by $t = 0, 1, 2, ..., \infty$. In the spirit of Lagos and Wright (2005), each time-period t is divided into two subperiods, labeled *day* and *night*. Households belong to one of two permanent groups: Group 1 and Group 2. Each group is of equal measure. Denote by A and B the set of Group 1 and Group 2, respectively.

All households meet at a central location during the day. Let $x_t(i) \in \mathbb{R}$ denote consumption (production, if negative) of output during the day by household $i \in A \cup B$ at date t. Linear preferences in $x_t(i)$ implies that utility is transferable. Assuming that the day good is perishable, an aggregate resource constraint implies

$$X \equiv \int_{A \cup B} x_t(i) di \leq 0 \tag{1}$$

for all $t \geq 0$.
Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production, respectively, output at night household $i \in A \cup B$ at date t. The utility from night consumption is given by $\delta_t u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. The utility from night production is given by $g(y_t(i))$, where $g' > 0$ for $y > 0$, $g'' \geq 0$. Following Kocherlakota (2003), we impose a spatial structure for night transactions, labeled *location 1* and *location 2*. After the shock to consumer type is realized, producers in group 1(2) households travel to location 2(1), while consumers in group 1(2) travel to location 1(2). This implies that a household cannot consume its own output at night. Perishability of the night good implies another aggregate resource constraint

$$\textstyle\int_A c_t(i) di \leq \int_B y_t(i) di \quad \text{and} \quad \int_B c_t(i) di \leq \int_A y_t(i) di. \tag{2}$$

For consumer heterogeneity, we introduce an idiosyncratic shock on consumer types that captures the differences in their marginal utilities. More specifically, at the beginning of each night, consumers experience an idiosyncratic preference shock represented by the parameter $\delta_t(i)$, where $\delta_t(i) \in \left\{\delta^l = 1, \delta^h = \eta\right\}$ and $\eta > 1$. The shock is *i.i.d.* across consumers within each group and across time. In this paper, we assume that the realization of these preference shocks δ_t is private information.

Households discount payoffs across period with the discount factor $\beta \in (0, 1)$, so that their preferences can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{x_t(i) + \delta_t(i) u(c_t(i)) - g(y_t(i))\right\}. \tag{3}$$

We focus on symmetric stationary allocations, where all agents are equally weighted and agents of the same type are treated the same and the two types are treated symmetrically. During the night, the social planner instructs consumers of a representative household to consume $c^j \in \{c^l, c^h\}$ conditional on type realization. Symmetric locations at night implies that there is a measure 1/4 of type h consumers, a measure 1/4 of type l consumers, and a measure 1/2 of producers; so that the resource constraint (2) can be expressed in another way

$$\frac{1}{4}c^l + \frac{1}{4}c^h = \frac{1}{2}y. \tag{4}$$

The ex-ante lifetime utility of households at a stationary allocation (c^l, c^h, y) is expressed as

$$W = \frac{1}{(1-\beta)}\left\{\frac{1}{2}\left[u(c^l) + \eta u(c^h)\right] - g(y)\right\}. \tag{5}$$

Linear utility in the day good x implies that any lottery over $\{x_t(i)\}$ for all $t \geq 0$ such that $E_0[x_t(i)] = 0$ can be a solution. A planner may set $x_t(i) = 0$ for all i households at date $t \geq 0$ as a trivial solution. The first-best allocation (c^{l^*}, c^{h^*}, y^*) maximizes (5) subject to the aggregate resource constraints (1) and (4).

The first-best allocation is characterized by

$$\begin{aligned} u'(c^{l^*}) &= \eta u'(c^{h^*}), \\ u'(c^{l^*}) &= g'(y^*), \\ c^{l^*} + c^{h^*} &= 2y^*. \end{aligned} \tag{6}$$

In what follows, we impose restrictions on this environment that will make a medium of exchange essential. We assume that households lack commitment and are anonymous. Limited commitment implies that all trades are voluntary satisfying sequential rationality (individually rational at every period t). Anonymity means that it is impossible to monitor the past action of agents pertaining to their trading histories. Given these assumptions, trade by credit is infeasible so that renders money—in the form of cash and CBDC—essential, as stated by Kocherlakota (1998). Furthermore, we restrict all trades to occur in a sequence of competitive spot markets with cash and CBDC being exchanged for goods in the day and night. This still preserves the essentiality of money even without search frictions as shown in Lagos and Wright (2005).

Money (or a medium of exchange) is essential because society would not be able to achieve desirable outcomes otherwise. Individuals would not be trading as a result of these frictions mentioned before. In contrast to Andolfatto (2011) where attention is restricted to linear mechanisms, we show how interest-bearing CBDC may improve welfare in nonlinear environments, much akin to Andolfatto (2010).

3 Decision-making of households

3.1 Government policy

The central bank and the government are a consolidated entity who issue intrinsically worthless tokens called cash and CBDC. Denote by $\{(v_1, v_2), (w_1, w_2)\}$ the price of cash and CBDC in the day and night, respectively. Let (M_c, M_e) denote the cash and CBDC supply for next period, which evolve over time, according to $M_c = \gamma_c M_c^-$ and $M_e = \gamma_e M_e^-$, respectively, where γ_c and γ_e denote (gross) growth rates of cash and CBDC, respectively (a superscript '-' stands for the previous period's cash and CBDC supply). The government's policy rule is to pay nominal interest rate R on CBDC balances to those individuals who are willing to pay the fixed fee, K. In addition, the government can also collect taxes, T_x, on labor x from each household that uses CBDC as a payment instrument during the day. This is because we are assuming that a household can hide cash balances to avoid paying the labor tax.

In what follows, the government has an aggregate interest obligation $(R-1)M_e$. The government can also print new money in the form of cash and CBDC $M_c - M_c^- + M_e - M_e^-$. The government can only collect the fixed CBDC fee $K^j \geq 0$ at night. A household enters the night market and decides whether to pay the fixed fee. If a household pays the fixed fee then he earns the nominal interest R at night from its CBDC holdings. Both K^j and R will affect the future CBDC balances carried forward into the next day. If a household declines to pay the fixed fee, then CBDC here works like cash.

We simplify matters by applying a result that holds in this class of quasilinear models. We design the government policy so that only the mass 1/4 of agents will voluntarily pay the fixed fee at night and conditional on their initial real CBDC balances a_e (explained in more details below) will pay the labor tax T_x in the day. Thus, a feasible government policy will have to satisfy the government budget constraint,

$$(R-1)M_e = M_c - M_c^- + M_e - M_e^- + T_x X 1_{a_e} + \frac{1}{4}K^j,$$

where $1_{a_e} \geq 0$ is an indicator function, for the labor tax that can be collected only when CBDC is used as a means of payments and X is the aggregate labor. By multiplying both sides of this latter expression by w_2, the government budget constraint may alternatively be expressed in real terms by

$$[(R-2)\gamma_e+1]\,w_2M_e^- - (\gamma_c-1)w_2M_c^- = \tau_x X + \frac{1}{4}\kappa^j, \tag{7}$$

where $\tau \equiv w_2 T$ and $\kappa^j \equiv w_2 K^j$. Furthermore, assume the cash and CBDC supply, respectivley, is expanded at a constant growth rate, that is, $\gamma_c \geq 1$ and $\gamma_e \geq 1$. We define a *zero intervention policy* as a policy when $\gamma_c = \gamma_e = R = 1$, so that $\tau_x = \kappa^j = 0$.

3.2 The day market

Households enter the day with $(m_1, e_1) \geq 0$ of nominal balances of cash and CBDC and the night market with $(m_2, e_2) \geq 0$. Households can trade x_c of the day good with cash and x_e of the day good with CBDC. The day budget constraint with cash is given by $x_c = v_1 m_1 - v_1 m_2$ and the day budget constraint with CBDC is given by $x_e = {}^{w_1 e_1}/_{1+T_x} - {}^{w_1 e_2}/_{1+T_x}$. Denote by $a_c \equiv v_1 m_1$ and $q_c \equiv v_2 m_2$ the real cash balances at day and the night, respectively. Similarly, denote by $a_e \equiv w_1 e_1$ and $q_e \equiv w_2 e_2$ the real CBDC balances at day and night, respectively. Define $\phi \equiv {}^{v_1}/_{v_2}$ and $\psi \equiv {}^{w_1}/_{w_2}$. Since $x = x_c + x_e$, the day-market budget constraint can now be expressed as

$$x = a_c - \phi q_c + \frac{w_2}{w_2+\tau_x} a_e - \frac{w_1}{w_2+\tau_x} q_e. \tag{8}$$

Denote by $W(a_c, a_e)$ the utility value of a household entering a day with real cash and CBDC balances, (a_c, a_e). Also denote by $V(q_c, q_e)$ the utility value of beginning the night with (q_c, q_e) cash and CBDC balances. Note that $V(q_c, q_e)$ denotes the value before a household knows its consumer type. The value functions W and V must satisfy the recursive relationship

$$W(a_c, a_e) \equiv \max_{q_c, q_e} \left\{ a_c - \phi q_c + \frac{w_2}{w_2+\tau_x} a_e - \frac{w_1}{w_2+\tau_x} q_e + V(q_c, q_e) \right\}. \tag{9}$$

We will later impose assumptions on V so the demand for both cash and CBDC are determined by the first-order conditions:

$$\frac{\partial V(q_c, q_e)}{\partial q_c} = \phi \tag{10}$$

and

$$\frac{\partial V(q_c, q_e)}{\partial q_e} = \frac{w_1}{w_2+\tau_x}. \tag{11}$$

Note that the demand for CBDC decreases with labor tax, τ_x (see also Gomis-Porqueras et al.

(2014)). Moreover, all households enter the night with identical real cash and CBDC balances $q_c \in (0,\infty)$ and $q_e \in (0,\infty)$. In other words, cash and CBDC demand are independent of the initial cash and CBDC holdings (a_c, a_e). This is often highlighted in the Lagos-Wright (LW) models. Applying the envelope theorem yields

$$\frac{\partial W(a_c, a_e)}{\partial a_c} = 1, \tag{12}$$

$$\frac{\partial W(a_c, a_e)}{\partial a_e} = \frac{w_1}{w_2 + \tau_x}. \tag{13}$$

3.3 The night market

A household carries over a nominal cash-CBDC portfolio of (m_2, e_2) at night. Consumer preference shock is realized at the beginning of the night market. Consumers and producers in a household travel to either *location 1* or *location 2*. A household makes the consumption and production decisions, which are carried out by consumers and producers by traveling into different locations.

Denote by $c^j = c_c^j + c_e^j$ for consumption of a household with realized consumer type j, where c_c^j is type j's consumption by using cash and c_e^j is type j's consumption by using CBDC. Similarly, $y^j = y_c^j + y_e^j$ is the output produced where a mixture of cash and CBDC can be used for its purchase. Hence, future cash balances are given by $m_1^+(j) = m_2 + v_2^{-1}(y_c^j - c_c^j)$. Expressed in real terms, this constraint is given by

$$a_c^+(j) = \frac{\phi}{\gamma_c}\left(q_c - c_c^j + y_c^j\right).$$

Future CBDC balances will depend on whether type j consumer will pay the fixed fee K^j to open a CBDC account with the central bank. A payment of K^j is required for a type j consumer to earn a nominal interest rate R. The interpretation of K^j and R is quite similar to that in Andolfatto (2010), except that these parameters are introduced into the night market after the realization of consumer types. Define the indicator function for a type j household by $\sigma^j \in [0,1]$, where $\sigma^j = 1$ means that a type j household pays the fixed fee at date t. In the event a household of type j opts not to pay the fixed fee, they do not accrue any interest on their CBDC holdings. In this scenario, CBDC functions similarly to cash, except that households pay the labor tax without earning any interest on their CBDC holdings. Given a type j household is willing to pay the fixed fee K^j, future CBDC balances are given by $e_1^+(j) = Re_2 - K^j + w_2^{-1}(y_e^j - c_e^j)$.[1] Alternatively, in real terms,

$$a_e^+(j) = \frac{\psi}{\gamma_e}\left(Rq_e - \kappa^j - c_e^j + y_e^j\right).$$

For a household with realized consumer type $j \in \{l, h\}$, there are two cash and CBDC constraints for consumption of night output; that is,

$$c_c^j \leq q_c, \tag{14}$$

$$c_e^j \leq \sigma^j(Rq_e - \kappa^j) + (1-\sigma^j)q_e. \tag{15}$$

Let $\lambda_c^j \geq 0$ and $\lambda_e^j \geq 0$ denote the Lagrange multipliers associated with the constraints (14) and (15), respectively. In what follows, the choice problem at night for a type j household is given by

$$\begin{aligned} V^j(q_c, q_e) \equiv \max_{\substack{\sigma^j, c_c^j, c_e^j, \\ y_c^j, y_e^j}} \Big\{ & \delta^j u(c^j) - g(y^j) \\ & + \beta W\left(\frac{\phi}{\gamma_c}\left[q_c - c_c^j + y_c^j\right], \frac{\psi}{\gamma_e}\left[\sigma^j(Rq_e - \kappa^j - c_e^j + y_e^j) + (1-\sigma^j)(q_e - c_e^j + y_e^j)\right]\right) \\ & + \lambda_c^j(q_c - c_c^j) + \lambda_e^j\left[\sigma^j(Rq_e - \kappa^j) + (1-\sigma^j)q_e\right]\Big\}. \end{aligned} \tag{16}$$

We now make the following assumptions on the function V:

Assumption 1 *i)* $V_{q_c, q_c} < 0 < V_{q_c}$ *and* $\phi < \frac{\partial V(0, q_e)}{\partial q_c}$;

ii) $V_{q_e, q_e} < 0 < V_{q_e}$ *and* $\frac{w_1}{w_2 + \tau_x} < \frac{\partial V(q_c, 0)}{\partial q_e}$;

iii) *V is non-differentiable at* $(R-1)q_e = \kappa^j$ *for* $\sigma^j \in [0, 1]$.

In fact, *Assumption 1* captures the properties of a stationary monetary equilibrium. By applying (13) and (14), all producers regardless of household types, produce identical output y_c with cash and y_e with CBDC satisfying

$$g'(y_c) = \frac{\beta\phi}{\gamma_c}, \tag{17}$$

$$g'(y_e) = \frac{\beta w_1}{\gamma_e(w_2 + \tau_x)}. \tag{18}$$

[1] Note that our CBDC fee structure has a nonlinear mechanism, which could be taken advantage of by a coalition of agents. However, the presumed absence of commitment makes such coalitions infeasible.

The demand for desired consumption c_c^j with cash and the desired consumption c_e^j with CBDC is characterized by the following first-order conditions:

$$\delta^j u'(c_c^j) = \frac{\beta\phi}{\gamma_c} + \lambda_c^j, \tag{19}$$

$$\delta^j u'(c_e^j) = \frac{\beta w_1}{\gamma_e(w_2+\tau_x)} + \lambda_e^j. \tag{20}$$

If the CBDC constraint binds ($\lambda_e^j > 0$), then a household with realized type j consumer will pay the fixed CBDC fee ($\sigma^j = 1$), as they would like to relinquish their money or discharge their debt for consumption the following day. But if the CBDC constraint is slack ($\lambda_e^j = 0$) then there could be three possiblities, so that the optimal decision to pay the fixed fee κ for a type j consumer satisfies

$$\sigma^j = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \text{if } \kappa^j \begin{cases} < \\ = (R-1)q_e. \\ > \end{cases} \tag{21}$$

As highlighted in Andolfatto (2010), only consumers with sufficiently large money holdings q_e will find it optimal to pay the fixed fee κ^j given that $R > 1$ and $\kappa^j > 0$. That is, CBDC is used in large-scale transactions and cash is used in small-scale transactions, as there is intrinsically no difference in how both these payment instruments are used if there is no fixed fee. Moreover, by the envelope theorem

$$\frac{\partial V^j(q_c,q_e)}{\partial q_c} = \frac{\beta\phi}{\gamma_c} + \lambda_c^j, \tag{22}$$

$$\frac{\partial V^j(q_c,q_e)}{\partial q_e} = \begin{cases} \frac{\beta R w_1}{\gamma_e(w_2+\tau_x)} + R\lambda_e^j \\ \\ \frac{\beta w_1}{\gamma_e(w_2+\tau_x)} + \lambda_e^j \end{cases} \text{if } \kappa^j \begin{cases} < (R-1)q_e \\ > (R-1)q_e. \end{cases} \tag{23}$$

Given the uncertainty of whether cash or CBDC will be utilized as a payment instrument, we can employ equations (17) and (18) to derive the following rate-of-return equality condition:

$$\frac{\phi}{\gamma_c} = \frac{w_1}{\gamma_e(w_2+\tau_x)}. \tag{24}$$

Condition (24) restricts attention to equilibria where cash and CBDC must have the same rate of return from the night to the next day, if they are to be accepted as payment. Alternatively, condition (24) can also be stated as a no-arbitrage condition. It follows that at the individual level, the cash-CBDC portfolio composition is indeterminate in equilibrium. We define $\zeta = \gamma_e/\gamma_c$ and rewrite this condition as

$$\zeta = \frac{w_1}{\phi\left(\frac{w_1}{\psi} + \tau_x\right)}. \tag{25}$$

The above condition ensures the co-existence of cash and CBDC, as they have the same rate of return. If $\zeta > w_1/\phi\left(\frac{w_1}{\psi}+\tau_x\right)$, then the rate of return on cash is higher than that of CBDC, so that all individuals will use cash. Conversely, if $\zeta < w_1/\phi\left(\frac{w_1}{\psi}+\tau_x\right)$ then all individuals will use CBDC.

Using (25) we can also derive an expression for the labor tax τ_x,

$$\tau_x = \frac{(\psi - \zeta\phi)\, w_1}{\zeta\phi\psi}. \tag{26}$$

If only cash is used by agents, then a tax on labor income is not attainable for the government, so $\tau_x = 0$. If CBDC is used then $\tau_x \geq 0$. We can obtain an upper bound on τ_x by defining $\overline{\tau_x} \equiv \tau_x < (\psi-\zeta\phi)w_1/\zeta\phi\psi$. Therefore, the range of values for labor tax is feasible within the interval $[0, \overline{\tau_x}]$, that is, $\tau_x \in [0, \overline{\tau_x}]$. Figure 1 depicts the cases in which cash and CBDC equilibria are separated by the labor tax τ_x. Note that $\zeta = 1$ implies $\phi = \psi$, so that the rate of return on cash and CBDC is exactly equal.

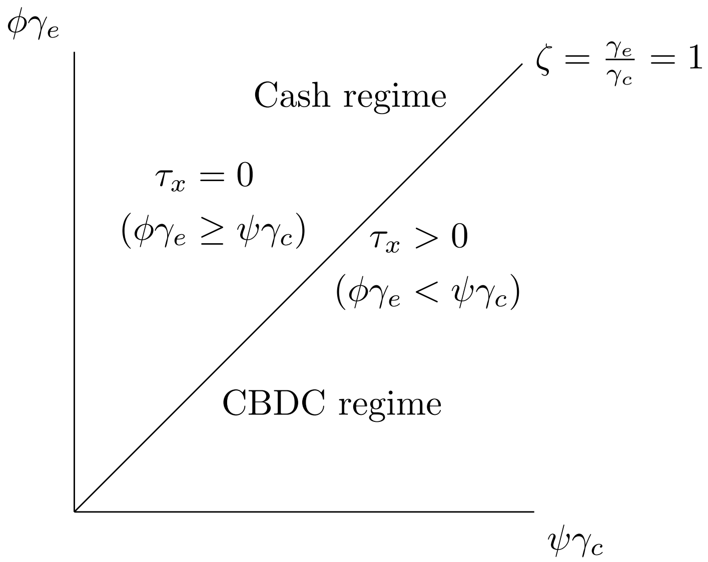

Figure 1: Separation of cash and CBDC equilbria

4 Competitive equilibrium

We now seek to characterize steady-state equilibria that can arise from the conditions in the model. As mentioned earlier, we can have economies where only cash is used, economies where only CBDC is used, and economies where a mixture of cash and CBDC is used by households.

Before deriving the equilibrium allocations, we first make some assumptions. On the one hand, since type h consumers receive a higher marginal utility from consumption, they are more likely to discharge their debt so that both their cash and CBDC constraints will bind, that is, $\lambda_c^h > 0$ and $\lambda_e^h > 0$. For this reason, we assume that type h consumers will pay the fixed fee $\kappa^h = \kappa$. On the other hand, type l consumers are likely to misrepresent themselves as type h consumers by hiding their money balances to acquire the high money balance at the beginning of period 0. For this reason, we can assume that their debt-constraints will remain slack, that is, $\lambda_c^l = \lambda_e^l = 0$. Therefore, type l consumers will not be paying the fixed fee, that is, $\kappa^l = 0$. Later on, we will consider the case where both the cash and CBDC constraints for type l consumers will bind.

4.1 CBDC-only economy ($\zeta < w_1/\phi\left(\frac{w_1}{\psi}+\tau_x\right)$)

4.1.1 Market clearing

Suppose that CBDC has a higher rate of return than cash, so that agents only use CBDC as a means of payment. We have two market-clearing conditions. For the night goods market, the clearing condition is (4). The market-clearing condition for the money market involving CBDC is given by

$$q_e = w_2 M_e^- . \tag{27}$$

4.1.2 CBDC Equilibrium

In what follows, we restrict attention to stationary equilibria; which entails $w_1/w_1^+ = w_2^+/w_2^+ = \gamma_e$. If the CBDC constraint for type h constraint binds then for any $\gamma_e > \beta$, we must have

$$c_e^h = Rq_e - \kappa. \tag{28}$$

First, note that since

$$\frac{\partial V(q_c,q_e)}{\partial q_e}=\frac{1}{2}\frac{\partial V^l(q_c,q_e)}{\partial q_e}+\frac{1}{2}\frac{\partial V^h(q_c,q_e)}{\partial q_e}, \tag{29}$$

combining (23) with $\sigma^l=0$ and $\sigma^h=1$ yields

$$\frac{\partial V(q_c,q_e)}{\partial q_e}=\frac{1}{2}\left[\frac{\beta w_1}{\gamma_e(w_2+\tau_x)}\right]+\frac{1}{2}\left[\frac{\beta R w_1}{\gamma_e(w_2+\tau_x)}+R\lambda_j^e\right]. \tag{30}$$

Combining (20) and (11) leads to

$$\frac{w_1}{w_2+\tau_x}=\frac{1}{2}u'(c_e^l)+\frac{1}{2}R\eta u'(c_e^h). \tag{31}$$

Now, combining (31) with the market clearing conditions (4) and (27) leads to

$$\left[2\left(\frac{\gamma_e}{R\beta}\right)-\frac{1}{R}\right]u'(c_e^l)=\eta u'(c_e^h). \tag{32}$$

Appealing to (18), one obtains

$$g'(y_e)=u'(c_e^l). \tag{33}$$

The equilibrium allocation $\left(c_e^l,c_e^h,y_e\right)$ for a CBDC economy is then characterized by (32), (33) and (4) when type l consumers are not debt constrained. Note that the "standard" Friedman rule prescription of setting $(R,\gamma_e)=(1,\beta)$ will result in the competitive monetary equilibrium corresponding to the first-best allocation. However, since type l consumers do not willingly pay the fixed fee κ, implementing a deflationary policy according to the Friedman rule is not feasible. Moreover, the labor tax τ_x does not affect the equilibrium allocation, as such a tax is voluntary in the sense that individuals can opt out of paying this tax by using cash instead of using CBDC for transactions. We have the following proposition.

Proposition 1 *When $\zeta < w_1/\phi\left(\frac{w_1}{\psi}+\tau_x\right)$, in a pure CBDC economy, the labor tax $\tau_x\in(0,\overline{\tau_x}]$ is non-distortionary and does not affect the equilibrium allocation $\left(c_e^l,c_e^h,y_e\right)$.*

4.2 Cash-only economy ($\zeta > {}^{w_1}/_{\phi\left(\frac{w_1}{\psi}+\tau_x\right)}$)

4.2.1 Market clearing

Assuming that the rate of return on cash is higher than that of CBDC, agents always demand physical currency and do not accept electronic means of payment. In addition to the night goods market clearing (given by condition (4)), the clearing condition for the money market in the form of cash is

$$q_c = v_2 M_c^-. \tag{34}$$

4.2.2 Cash Equilibrium

Similar to the CBDC-only economy, we focus on stationary equilibria in a cash-only economy; in which case ${}^{v_1}/_{v_1^+} = {}^{v_2^+}/_{v_2^+} = \gamma_c$. Assuming that the cash constraint for type h binds, we will have $c_c^h = q_c$ for any $\gamma_c > \beta$. Once again, since

$$\frac{\partial V(q_c, q_e)}{\partial q_c} = \frac{1}{2}\frac{\partial V^l(q_c, q_e)}{\partial q_c} + \frac{1}{2}\frac{\partial V^h(q_c, q_e)}{\partial q_c}, \tag{35}$$

combining (10), (19), and (22) yields

$$\phi = \frac{1}{2}u'(c_c^l) + \frac{1}{2}\eta u'(c_c^h). \tag{36}$$

Combining (19) when type l consumers are not cash constrained with (36) along with the market-clearing conditions (4) and (34) leads to

$$\left[2\left(\frac{\gamma_c}{\beta}\right) - 1\right] u'(c_c^l) = \eta u'(c_c^h). \tag{37}$$

Considering (17), one obtains

$$g'(y_c) = u'(c_c^l). \tag{38}$$

Now, the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy is characterized by (37), (38) and (4). Once again, while the Friedman rule ($\gamma_c = \beta$) could potentially result in the first-best allocation, its implementation is unfeasible in this economy due to the lack of a lump-sum tax instrument associated with cash. This limitation arises from the fact that individuals have the option to conceal their cash balances should they desire to consume more of the day good in the following day.

The above equilibrium allocation is assuming that the cash constraint for type l consumers is slack. We now consider when type l consumers are cash constrained, that is, $\lambda_c^l > 0$. Using (14) and the market-clearing condition (4), one obtains

$$c_c^l = c_c^h = y_c. \tag{39}$$

Furthermore, combining (17) and (36) gives rise to

$$g'(y_c) = \frac{\beta}{2\gamma_c}(1+\eta)\,u'(y_c). \tag{40}$$

The equations (39) and (40) fully characterize the equilibrium allocation $\left(c_c^l, c_c^h, y_c\right)$ for a cash economy when the debt-constraints for both type l and type h consumers bind. As the night good y_c and the ex ante welfare W are strictly decreasing in γ_c in both these scenarios, the optimal policy is to set a zero intervention policy with a fixed money supply.

Proposition 2 *When $\zeta > \psi/\phi$ with $\tau_x^* = 0$, in a pure cash economy, the optimal policy is to set $\kappa^* = 0$, which implies $\gamma_c^* = 1$.*

Owing to the limited commitment and anonymity that make lump-sum taxation unfeasible, setting the lower limit to $\gamma_c = 1$ optimizes welfare. The second-best solution, as outlined in Xiang (2013), is to maintain a passive policy that minimizes inflation and simultaneously maximizes the rate of return on currency.[2]

4.3 Redistributive policy with CBDC

In the event that the CBDC constraint for type l consumers is not binding, a sufficiently low rate of inflation will be necessary to redirect purchasing power in a socially favorable manner. This is because type l saving falls as inflation rate rises. Since inflation effectively serves as a tax on currency, the purchasing power of the unconstrained type l consumers diminishes when they face higher inflation. Depending on the specific parameters, these unconstrained l types could become constrained and that would not decrease their saving.

To this end, we consider a mixed cash and CBDC economy with a policy of zero intervention; so that $\tau_x = \kappa = 0$ and $\gamma_e^1 = \gamma_c^1 = R = 1$. Using condition (32), the CBDC constraint for both

[2] Indeed, this discovery closely aligns with Proposition 1 in Xiang (2013).

types of consumers will bind when $\beta < 2/(1+\eta)$ conditional on $\eta_0 \equiv \eta > 1$ and $\gamma_e^1 = \gamma_c^1 = 1$ and $R^1 = 1$. As in Andolfatto (2011), we refer to this economy as an impatient economy. Then along with another market-clearing condition for the mixed regime $q_c = w_2 M_c^-$, the equilbrium allocation must satisfy (28) and

$$c_e^l = q_e. \tag{41}$$

Note that first we have to solve for the equilibrium fixed fee, κ, to simplify the equations above. Making use of the government budget constraint (7) and combining with (27), we can obtain

$$\kappa = 4\{[(R-2)\gamma_e + 1]\, q_e - \tau_x X\}. \tag{42}$$

Applying the aggregate resource constraint (1), the latter expression can be further reduced to

$$\kappa^* = 4\{[(R-2)\gamma_e + 1]\, q_e\}. \tag{43}$$

Condition (43) is the optimal κ^* that is necessary to make the debt-constraints bind for both types of consumers. Furthermore, the linear utility in the day good x yields a result that is immediately apparent.

Proposition 3 *κ^* is determined independently of τ_x.*

In other words, given that agents have linear preferences in $x_t(i)$, they are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a specific expected value. As a result, the fixed CBDC fee is not dependent on the labor tax, τ_x. Though the labor tax does not directly generate revenue for the government, it serves as a device to influence individual behavior.

Next, we will derive the equilibrium allocation when agents use CBDC. Using the night goods market-clearing condition (4) along with (28) and (41), this implies

$$\frac{1}{4} q_e + \frac{1}{4}(R q_e - \kappa) = \frac{1}{2} y;$$

so that

$$q_e = \frac{2\zeta\phi\psi y_e}{\zeta\phi\psi\{1 + R - 4[(R-2)\gamma_e + 1]\}}. \tag{44}$$

Now, plugging back everything, the equilibrium allocation is, in this case, given by

$$c_e^l = \frac{2\zeta\phi\psi y_e}{\zeta\phi\psi\{1+R-4[(R-2)\gamma_e+1]\}}, \tag{45}$$

$$c_e^h = \frac{2R\zeta\phi\psi y_e - 8[(R-2)\gamma_e+1]\zeta\phi\psi y_e}{\zeta\phi\psi\{1+R-4[(R-2)\gamma_e+1]\}}. \tag{46}$$

From the above conditions, we can ensure that the saving of type l consumers, $s_e^l(\gamma_e,\gamma_c,R) \equiv q_e(\gamma_e,\gamma_c,R) - c_e^l(\gamma_e,\gamma_c,R)$, goes to zero if the CBDC constraint for l types binds. Alternatively, for a binding cash constraint, type l saving $s_c^l(\gamma_e,\gamma_c,R) \equiv q_c(\gamma_e,\gamma_c,R) - c_c^l(\gamma_e,\gamma_c,R) = 0$ from condition (39).

We now consider an economy that is sufficiently patient, that is, $\beta \geq 2/(1+\eta)$. In this scenario, with sufficiently low inflation and high nominal interest rates, the cash and CBDC constraints for the l types continue to be slack. Under these conditions, the equilibrium allocation in a mixed cash-CBDC regime is characterized by (32) and (37). This means that inflation hurts efficiency as long as both the cash and CBDC constraints remain slack. It is easy to verify that $s_c^l(\gamma_e,\gamma_c,R) \equiv q_c(\gamma_e,\gamma_c,R) - c_c^l(\gamma_e,\gamma_c,R)$ is monotonically decreasing in γ_c and that $s_c^l(\gamma_e,\gamma_c,R) = 0$ for some $\gamma_c^0 \geq \gamma_c^1$. Considering type l saving with CBDC, we have

$$s_e^l(\gamma_e,\gamma_c,R) = \frac{2y_e + \kappa(\gamma_e,\gamma_c,R) - (1+R)c_e^l(\gamma_e,\gamma_c,R)}{R}. \tag{47}$$

For $\beta > 2/(1+\eta)$, we know that $s_e^l(1,1,1) = 2y_e > 0$. Note that $s_e^l(\gamma_e,\gamma_c,R)$ is increasing in κ. The policy parameters (γ_e,γ_c,R) will have an indirect effect on type l saving with CBDC. Observe that by condition (32), $c_e^l(\gamma_e,\gamma_c,R)$ is monotonicaly increasing in γ_e and monotonically decreasing in R. However, the indirect effect of κ on s_e^l is difficult for us to establish. This is because the effect of these policy parameters on the fixed fee κ is ambiguous. If κ were decreasing (increasing) in γ_e (γ_c) and increasing in R, then $s_e^l(\gamma_e,\gamma_c,R)$ would be decreasing (increasing) in γ_e (γ_c) and increasing in R. If that were to be the case, then there would exist a CBDC inflation rate $1 < \gamma_e^0 < \infty$, a cash inflation rate $1 < \gamma_c^0 < \infty$ and a nominal interest rate $R^0 > 1$ such that $s_e^0(\gamma_e^0,\gamma_c^0,R^0) = 0$. This would guarantee that the optimal transfer creates an inflation rate that is high enough and a nominal interest rate that is low enough to make the type l consumers debt-constrained. However, it ensures that the inflation and nominal interest rates are not too excessive to deter the consumers from accumulating an adequate amount of real cash and CBDC balances, denoted by q_c and q_e, respectively.

When $\beta < 2/(1+2\eta)$, payment of fixed fee κ by type h consumers will increase their purchasing power when both consumer types are debt constrained. This is assuming $\psi \geq \phi\zeta$, a condition that will guarantee the coexistence of CBDC alongside cash as they have the same rate of return. Given this assumption, type h consumers would be using CBDC and type l

would be using cash. The interesting question is how to induce type l consumers to switch to CBDC and extract the fixed fee κ. When $\beta \geq 2/(1+2\eta)$, type l consumers would be willing to pay the fixed fee if the saving from CBDC is at least as high as the saving from cash, that is, $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$. We summarize our argument by stating the following conjecture.

Conjecture 1 *When $\beta < 2/(1+2\eta)$, the optimal policy is characterized by $(\gamma_c^*, \gamma_e^*) > 1$ that satisfies (39), (40), (43), (45), and (46) in a mixed cash-CBDC economy with the condition $\psi \geq \phi\zeta$. When $\beta \geq 2/(1+2\eta)$, the optimal policy must satisfy $\gamma_c^* \geq \gamma_c^0$, $\gamma_e^* \geq \gamma_e^0$ and a nominal interest rate $R^0 \geq R^1$ so that $s_e^l(\gamma_e^1, \gamma_c^1, R^1) \geq s_c^l(\gamma_e^1, \gamma_c^1, R^1)$, with an equilibrium allocation characterized by (32), (33), (37), and (38). For both these conditions, $\tau_x \in [0, \overline{\tau_x}]$ as defined by (26).*

In the conjecture, a binding CBDC constraint for the l types results in the net positive revenue for the government, as the fixed CBDC fee and the labor tax can be positive. Since the labor tax in the CBDC economy is non-distortionary, the introduction of a CBDC may improve welfare. This is because with cash, the labor tax is not feasible as agents can hide their money balance to evade taxes. If CBDC has to have a higher rate of return than cash, then CBDC inflation will need to be lower than cash inflation. Since $\beta \geq 2/(1+\eta)$, a relatively lower inflation rate with CBDC than with cash ($\gamma_e^0 \leq \gamma_c^0$) can increase the saving for l types if they use CBDC instead of cash. By potentially attaining a higher equilibrium level of output y, a CBDC economy can improve welfare compared to a cash economy.

5 Conclusion

We study how the issuance of CBDC might impact welfare in an economy where tax evasion through the usage of cash is possible. Our model incorporates private information to introduce heterogeneity among agents. A crucial finding is that CBDC can benefit those individuals who have an immediate need for consumption when their debt-constraints are binding. The labor tax, which only applies when CBDC is used as a payment instrument, poses no distortion, as individuals can conceal their cash balances. While the government might face challenges financing the cost of CBDC if those with a lower marginal utility are unwilling to pay the CBDC fee, the labor tax itself does not introduce any distortion.

A critical component of our model is the requirement for all exchanges to be voluntary, which naturally limits policies to those respecting individual rationality. Low inflation policies can be achieved if individuals with a lower marginal utility of consumption are incentivized to contribute to the government's CBDC expenditure. Given that inflation with cash is relatively higher than inflation with CBDC, leveraging the labor tax to finance CBDC expenditure presents a valuable opportunity for the government. In other words, the introduction of CBDC can enhance welfare and potentially lead to a more efficient allocation of resources by deterring tax evasion.

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